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Model for optimizing thermal efficiency of open indirect cycle of gas turbine based on the mathematical programming

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ABSTRACT

Open indirect cycle of gas turbine is almost similar to a simple Brayton cycle but it can be used for heat recovery means in different industries, specially, in nuclear power plants. In this investigation, a model of optimal thermal efficiency of open indirect cycle of gas turbine has been developed and it has been applied for heat recovery from a nuclear reactor. The model may be categorized as a nonlinear mathematical programming model and the objective function is maximization of thermal efficiency in a given site conditions. Total compression ratio, cycle temperature ratio and cycle pressure ratio are identified as three dimensionless numbers and they have been considered as basic decision variables. The model has been applied to identify the optimal design condition of the heat recovery system.

1. INTRODUCTION:

Fuel consumption in industries contributes to the emission of greenhouse gases in general and CO_2 in particular. Emission of CO_2 is a function of technical operation of the industries and the waste energy flow. Improvement in energy efficiency and the design of technologies has reduced wasting energy of industries considerably over last three decades. But rapid economic growth in many developing countries and increased welfare of population in industrialized nations has led to expansion of industrial factories world wide. Number of factories has increased considerably and air pollution is observed in major cities in the world, including both developing and industrialized countries.

Energy recovery is considered as a sufficient method for reducing air pollution due to converting waste energy to useful work in different industries. It may be categorized as preheating of combustion air or fuel in industrial furnace, preparing hot water in ventilation systems, producing superheated steam in boilers or generating additional electrical power in industries [1].

Open indirect cycle of gas turbine is one of the general forms of heat recovery systems which may be used to recovery waste heat from cooling circuit of nuclear reactors [2]. Its cycle is similar to a simple Brayton cycle where combustion chamber is replaced with a recuperator in order to heat recovery from coolant fluid in nuclear reactor cooling circuit. Absorbed heat is then used for increasing turbine inlet temperature and generating of additional power in nuclear power plant [3].

Many analytical tools have been developed for modeling open indirect cycle of gas turbine and its contribution to the implementation of the objectives of heat recovery. Some kinds of methods that have been suggested are the thermodynamically orientated simulators [4-12]. They study operation of the system based on the predefined design conditions.

Identification of optimal thermal efficiency of the open direct cycle of gas turbine has been the subject of the present work. A model of nonlinear mathematical programming has been developed and it has been applied for identifying the appropriate means of achieving the objectives of optimal design of the cycle. Theoretical concept of the model shall be presented in the next section and it shall be followed by demonstration of the application of model for defined case study.

2. THEORETICAL CONCEPT:

The model has been found on the thermodynamics theory and it has been tailored to identify the optimal design condition of specified heat recovery technology. The objective of the model has been to maximize the total thermal efficiency of an open indirect cycle of gas turbine which is used for heat recovery from coolant flow of cooling circuit of a nuclear reactor. Implementation of this concept has help to identify optimal design condition while the technology is considered as fixed.

Figure (1) shows the system of heat recovery. It contains two stages compressors with one intercooler between them, one recuperator for absorbing waste heat from coolant flow, one gas turbine which is coupled with a generator for producing additional electricity in nuclear power plant.

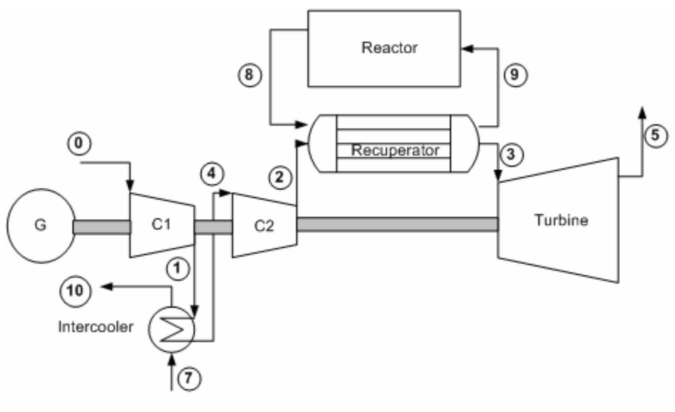

Figure 1- Heat recovery system Schematic

Total thermal efficiency in the Brayton cycle (η_{th}) may be defined as the ratio of net output power (W_{net}) to total input or absorbed heat (Q_{add}) as follows:

$$\eta_{th} = \frac{W_{net}}{Q_{add}} \tag{1}$$

According to figure (1), net output power and total absorbed heat can be calculated by the help of following equations:

$$W_{net} = m^0 C_p \{(T_3 - T_5) - (T_2 - T_4) - (T_1 - T_0)\} \tag{2}$$

$$Q_{add} = m^0 C_p (T_3 - T_2) \tag{3}$$

If the heat capacity of working fluid is nearly considered constant through the cycle, thermal efficiency of cycle should be rearranged as follows:

$$\eta_{th} = \frac{(T_3 - T_5) - (T_2 - T_4) - (T_1 - T_0)}{(T_3 - T_2)} \tag{4}$$

When the compressor unit is considered as an adiabatic system, dimensionless compression ratio for each of compressors may be represented by equation (7) through using equations (5) and (6):

$$T_1 - T_0 = T_0 (\frac{T_1}{T_0} - 1) \tag{5}$$

$$\frac{T_1}{T_0} = \left(\frac{P_1}{P_0}\right)^{\left(\frac{\gamma-1}{\gamma\eta_p}\right)} \tag{6}$$

$$Cc_1 = r_{C1}^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_1}{T_0} = Cc_1^{1/\eta_p} \tag{7}$$

Efficiency of first stage compressor may be defined by the help of equation (8):

$$\eta_{C1} = \frac{Cc_1 - 1}{Cc_1^{1/\eta_p} - 1} \tag{8}$$

Substituting equations (8) and (7) into equation (5) yields:

$$T_1 - T_0 = \frac{T_0}{\eta_{c_1}} (Cc_1 \ \ -1) \tag{9}$$

And it will be obtained for second stage compressor similar to first stage compressor:

$$T_2 - T_4 = \frac{T_2}{\eta_{c_2}} (Cc_2 \ \ -1) \tag{10}$$

With following stream flow from inlet of first stage compressor to outlet of second stage compressor, dimensionless total compression ratio can be defined as follows:

$$P_1 = (P_0 - \beta_1) r_{c1} \tag{11}$$

$$P_2 = (P_1 - (\beta_2 + \Delta P_{inc})) r_{c2} \tag{12}$$

$$Cc_T = \frac{(P_0 - \beta_1) Cc_1^{\frac{\gamma}{\gamma-1}} - (\Delta P_{\text{int}} + \beta_2) Cc_2^{\frac{\gamma}{\gamma-1}}}{P_0} \tag{13}$$

When compressed flow enters to the turbine, its pressure will be reduced. It will be clear that the outlet pressure of the turbine should never reach to atmospheric pressure. A dimensionless parameter is introduced to control this ratio:

$$L = \frac{P_5}{P_0} \tag{14}$$

According to equation (14), the dimensionless expansion ratio of gas turbine can be represented by following equations:

$$C_t = \left(\frac{P_3}{P_0}\right)^{\frac{\gamma-1}{\gamma}} \tag{15}$$

$$C_t = \left(\frac{P_3}{P_5 L}\right)^{\frac{\gamma-1}{\gamma}} \tag{16}$$

$$C_t = (r_t L)^{\frac{\gamma-1}{\gamma}} \tag{17}$$

$$C_t = \frac{r_t^{\frac{\gamma-1}{\gamma}}}{x} \tag{18}$$

Where, $x = (\frac{1}{L})^{\frac{\gamma-1}{\gamma}}$ and $r_t = \frac{P_3}{P_5}$.

Turbine stage efficiency then can be calculated by equation (19):

$$\eta_t = \frac{1-\dfrac{1}{C_t x^{\eta_p}}}{1-\dfrac{1}{\dfrac{1}{C_t x}}} \tag{19}$$

According to the principles, difference between P_5 and P_0 should be negligible and also, air side pressure drop of recuperator should not be allowed to be high, therefore, following relationship may be considered between outlet and inlet pressures ratio through recuperator and turbine:

$$\frac{P_3}{P_2} = \frac{P_0}{P_5} \tag{20}$$

According to the above assumption, x can be rewrite as below:

$$x = \left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_3}{P_0 Cc_T^{\frac{\gamma-1}{\gamma}}}\right)^{\frac{\gamma-1}{\gamma}} = \frac{C_t}{Cc_T} \tag{21}$$

According to the principles, difference between P_5 and P_0 should be negligible and also, air side pressure drop of recuperator should not be allowed to be high, therefore, following relationship may be considered between outlet and inlet pressures ratio through recuperator and turbine:

$$\frac{P_3}{P_2} = \frac{P_0}{P_5} \tag{20}$$

It can be concluded that equation (21) manifests the relationship between variations of turbine expansion ratio based on the compressors compression ratio. Therefore, x is named dimensionless cycle pressure ratio. It is also evident that x should be set at the right level if the thermal efficiency is to be maximized. Hence, identification of the relationship between C_t and Cc_T is an important factor of the aforementioned system design.

Respectively, temperature difference between inlet and outlet streams of the turbine may be represented by following equation:

$$T_3 - T_5 = T_3 \eta_t \left(1 - \frac{1}{Ct.x}\right) \tag{22}$$

According to the principles of gas turbine design, maximum power which may be obtained from the turbine is strictly affected by the turbine inlet temperature (T_3) [13]. This parameter will be influenced by the amount of recoverable heat through using the recuperator. Therefore, dimensionless cycle temperature ratio (t) is introduced in the model on the basis of the Carnot efficiency definition for controlling ratio of the maximum turbine inlet temperature to ambient temperature:

$$t = \frac{T_3}{T_0} \tag{23}$$

Dimensionless cycle temperature ratio may be formed as a function of technical indicators of the cycle if T_3 is replaced by following equations:

$$\varepsilon_H = \frac{T_{hin} - T_{Cin}}{T_{cout} - T_{Cin}} \tag{24}$$

$$T_3 = (1-\varepsilon_H) T_2 + \varepsilon_H T_8 \tag{25}$$

$$t = (1-\varepsilon_H)\frac{T_2}{T_0} + \varepsilon_H \frac{T_8}{T_0} \tag{26}$$

$$T_4 = (1-\varepsilon_{inc}) T_1 + \varepsilon_{inc} T_7 \tag{27}$$

$$T_2 = T_4 \left(1 + \frac{Cc_2 - 1}{\eta_{c2}}\right) \tag{28}$$

$$\begin{aligned} t = {} & (1-\varepsilon_H)(1-\varepsilon_{inc}) Cc_1 \left(1 + \frac{Cc_2 - 1}{\eta_{C2}}\right) + \\ & ((1-\varepsilon_{inc})\varepsilon_{inc}\left(1 + \frac{Cc_2 - 1}{\eta_{C2}}\right) + \varepsilon_H \frac{T_8}{T_0} \end{aligned} \tag{29}$$

The total net output power can be determined from equation (30):

$$W_{net} = m^0 Cp T_0 DSW \tag{30}$$

Where, DSW is defined as dimensionless specific work of cycle. It will be calculated by substituting equations (9), (10), (22), (25), (27) and (28) into equation (2) as follow:

$$DSW = t\eta_t \left(1 - \frac{1}{C_t.x}\right) - (1-\varepsilon_{\text{int}}) Cc_1 \left(\frac{Cc_2 - 1}{\eta_{c2}}\right) - \varepsilon_{\text{int}} \left(\frac{Cc_2 - 1}{\eta_{c2}}\right) \tag{31}$$

Finally, thermal efficiency of open indirect cycle of gas turbine may be represented by means of equation (32):

$$\eta_{th} = \frac{t\eta_t\left(1 - \dfrac{1}{C_t.x}\right) - (1-\varepsilon_{\text{int}})Cc_1\left(\dfrac{Cc_2 - 1}{\eta_{c2}}\right) - \varepsilon_{\text{int}}\left(\dfrac{Cc_2 - 1}{\eta_{c2}}\right) - \dfrac{Cc_1 - 1}{\eta_{c1}}}{t - \left[(1-\varepsilon_{\text{int}})Cc_1\left(1 + \dfrac{Cc_2 - 1}{\eta_{c2}}\right)\right] - \left[\varepsilon_{\text{int}}\left(1 + \dfrac{Cc_2 - 1}{\eta_{c2}}\right)\right]} \tag{32}$$

Equation (32) reveals that dimensionless numbers such as cycle temperature ratio, total compression ratio and cycle pressure ratio and also efficiency of compressors and effectiveness coefficients of intercooler and recuperator influence the thermal efficiency of open indirect cycle of gas turbine through using it as a heat recovery system.

3. MATHEMATICAL MODELING APPROACH:

The model is formulated as a constrained optimization programming with nonlinear objective function and constraints (NLP)[1]. GAMS/CONOPT is used to solve the NLP problem though following format [14]:

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[1] - Nonlinear Programming

$$Max\ Z = f(x_1,....,x_n) \quad (33)$$

Subject to:

$$g_i(x_1,...,x_n) \le b_i \quad (34)$$

$$x_j \ge 0 \quad (35)$$

And

$i = 1,2,3,...,m$

$j = 1,2,3,...,n$

Where, Z is defined as objective function and x and b are represented as decision variables and upper limits of constraints.

According to this mathematical approach, variables, parameters and constraints are categorized as follows:

3.1. Decision variables:

1) Cc_1, Cc_2 and x are defined as independed variables.

2) t, Ct, η_t, η_{C1} and η_{C2} are defined as first order depended variables.

3) Temperatures, Pressures and flow rate of working fluid are defined as second order depended variables.

3.2. Exogenous Parameters

Temperature, Pressure and flow rate of coolant fluid which its heat must be recovered by working fluid and also ambient temperature and pressure are defined as exogenous parameters of the model.

3.3. Technical Constraints:

Set of equations (8), (9), (10), (13), (14), (18), (19), (21), (22), (25), (27), (28), (29), and (31) are considered as technical constraints of the model. The limitations of Cc and x are included according the design condition:

$$1.1 < Cc < 2 \quad (36)$$

$$x < 1 \quad (37)$$

3.4. Objective function

Objective function (Z) describes degree of utilization of the system. In this model, thermal efficiency is defined as objective function by the help of equation (32) which should be maximized subject to satisfying technical constraints.

4. RESULTS AND DISCUSSION:

The model that has been developed in the present work has been applied for analysis of open indirect cycle of gas turbine as a heat recovery system. A case study has been designed on the basis of heat recovery from the outlet coolant flow of cooling circuit of a nuclear reactor.

Parameters of the abovementioned case study are described in table (1). The optimum results of the model can be observed in table (2).

Table 1- parameters of the case study

Parameter	**Unit**	**Value**
Outlet Coolant flow Temperature	C	800
Outlet Coolant flow pressure	Bar	1.1
Ambient temperature	C	25
Ambient Pressure	Bar	1

Table 2- Optimum results of the model

Item	**Unit**	**Value**
η_{th}	%	26.8
η_C	%	80.2
η_T	%	93.5
Cc		1.49
Ct		2
t		3.401
T_1	C	184.396
T_2	C	208.862
T_3	C	740.402
T_4	C	60.940
T_5	C	271.273
P_1	bar	3.489
P_2	bar	11.791
P_3	bar	11.314
P_4	bar	3.4
P_5	bar	1.306
r_c		3.489
r_t		10.923
Cc_T		2.020
x		0.99
ε_H		0.9
ε_{inc}		0.9

4.1. Impact of dimensionless total compression ratio

Figure (2) shows variation of the thermal efficiency with the dimensionless total compression ratio. It is clear that thermal efficiency will be increased in simple Brayton cycle with increasing of compression ratio of compressor. It can be observed in this figure that the thermal efficiency is increased to its maximum point at each of selected values for x from 0.97 to 0.99 and after that, it will be decreased. Thermal efficiency reaches to the optimum point of 0.268 at Cc_T =2.020. It can be observed that the main difference between simple Brayton cycle and open indirect cycle is the regressive trend of thermal efficiency after its optimum point due to replacing combustion chamber with recperator. Also, the decision variable (x) has direct effect on improving thermal efficiency due to increasing dimensionless specific work of cycle.

4.2. Impact of dimensionless cycle temperature ratio

Variation of thermal efficiency with dimensionless total temperate ratio is depicted in figure (3). The figure represents that the thermal efficiency increases smoothly until its maximum point at t=3.401 and x=0.99 and then decreases gradually. Unlike simple Brayton cycle, maximum thermal efficiency will not be obtained at maximum turbine inlet temperature. It may be explained by the fact that the turbine inlet temperature will be limited by the amount of waste heat and ability of the recperator for recovering it.

Whereas, the potential of heat source is constant, increase in turbine inlet temperature (T_3) necessitates increase in inlet temperature of working fluid into recuperator (T_2) and consequently increase in power consumption for two stages compressors. Therefore, after the optimum point, thermal efficiency will be reduced when the turbine inlet temperature or the dimensionless cycle temperature ratio is increased.

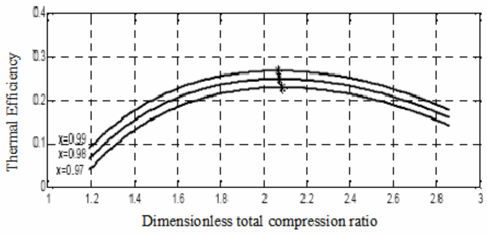

Figure 2. Variation of thermal efficiency with dimensionless total compression ratio

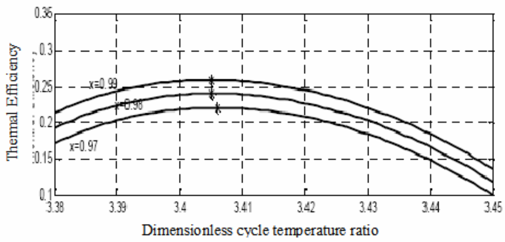

Figure 3. Variation of thermal efficiency with dimensionless total temperature ratio

4.3. Impact of dimensionless cycle pressure ratio

It can be observed from figures (2) and (3) that maximum thermal efficiency will be obtained at maximum dimensionless cycle pressure ratio ($x = 0.99$). The results of the model indicate that when dimensionless cycle pressure ratio is increased, it will be possible to obtain maximum point of thermal efficiency at lower value of dimensionless total compression ratio (Cc_T) and cycle temperature ratio (t). It means that a correct estimation of x is preferred which is compatible with the objective of the economic designing of the heat recovery system.

5. CONCLUSION

The objective of this research work has been to introduce an optimal designing approach of open indirect cycle of gas turbine as a heat recovery system based on the maximization thermal efficiency. The results indicate that implementation of the optimal designing approach based on coordination of dimensionless parameters such as total compression ratio, cycle temperature ratio and cycle pressure ratio through operation mode would lead to maximum of thermal efficiency. The results show that maximum thermal efficiency of the system can be obtained with $Cc_T = 2.020$, $t = 3.401$ and $x = 0.99$ at 26.8%.

The results of the model indicate that compression ratio and turbine inlet temperature increase thermal efficiency until certain point and after that, increment of these two factors reduces thermal efficiency through replacing combustion chamber with recuperator.

Application of the model results would enable to identify maximum thermal efficiency at maximum allowable value of dimensionless cycle pressure ratio. It means that, dimensionless expansion ratio of turbine should be estimated as much as closest to dimensionless total compression ratio. It provides an alternative approach to predefined thermodynamical simulations with the objective of the economic designing for the similar systems.

NOMENCLATURE

Symbol	Quantity	SI Unit
C_p	Heat capacity	kJ/kgC
C_t	Dimensionless expanssion ratio	
Cc	Dimensionless compression ratio	
Cc_T	Dimensionless total compression ratio	
m^0	Flow rate of working fluid	Kg/s
P	Pressure of working flow	bar
r_c	Compression ratio of each stage	
r_t	Expansion ratio of turbine	
T	Temperature	C
γ	Heat capacity ratio	
η_p	Polytrophic efficiency of compressor	
η_{th}	Thermal effeicincey of cycle	
β	Pressure drop at the inlet section of compressor	bar
ΔP_{inc}	Pressure drop of intercooler	bar
ε	Effectiveness coefficient	

Subscripts	Quantity
hin	Inlet hot stream into recuperator
cout	Outlet cold stream from recuperator
cin	Inlet cold stream into recuperator
inc	Intercooler
H	Recuperator

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